Constructing Intelligence in Point-to-Point Mobility Systems

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***Abstract*— Mobility on Demand (MoD) systems allow users to pick-up and drop-off vehicles (bikes, automobiles) ubiquitously in networks of parking stations. Asymmetric trip patterns cause imbalanced fleet allocation decreasing level of service. Current redistribution policies are complex to plan and typically cost more than the usage revenues of the system. This paper discusses a new operation model based on a double auction market where cost-minimizing users are both buyers and sellers of trip rights while profit-maximizing stations are competing auctioneers that trade them. Thus, trips are priced relatively to the inventory needs of origin and destination stations, causing some trips to be more expensive while other trips to pay back.**

Keywords-component; mobility-on-demand; vehicle sharing; dynamic pricing; self-organization

I. INTRODUCTION

In *Reinventing the Automobile*, William Mitchell, Chris Borroni-Bird, and Lawrence Burns describe their vision on the future of intelligent mobility: cities covered with dense networks of charging stations and shared fleets of electric vehicles allowing users to make point-to-point trips on demand [1]. A layer of sensors, communication networks, computing units, and mobile devices allows users to quickly locate available vehicles or parking docks in real-time while operators to decide any necessary modifications in the system. What is somehow unclear in this description is how information from the physical world turns into action. Who senses the world, who distributes information, who makes decisions, who takes actions, and who evaluates the results? How is intelligence constructed? We discuss recent work carried through at the Smart Cities and Changing Places groups of the MIT Media Lab during the period 2008-2011 on a new operation model for Mobility on Demand (MoD) systems based on self-organization [2]. The structure of the paper is organized as follows. The first part provides an overview on MoD systems using bike sharing as a case study. The second part reviews prior work in the field. The third part discusses the design of the Market Economy of Trips (MET) a new operation model for intelligent MoD systems. The fourth part discusses the equilibrium conditions of the MET. Finally, we conclude with the future directions of this work.

A. MoD and Bike Sharing Systems

Bike sharing systems (BSS) are gaining increasing attention in the research communities of intelligent MoD transportation systems. They constitute watertight systems to model, provide plenty of real-time data to use, and pose some interesting yet challenging research problems. Currently, more than 300 BSS around the world mobilize more than 3 million trips across 15,000 stations every day. Despite their convenience, BSS systems have significant operational limitations. About 10%-40% of the daily trip volume remains imbalanced due to asymmetric trip patterns. This displaced fleet must be hauled back by the end of each day otherwise the system starts next morning in a worse condition. To address asymmetric trip patterns, cities like Paris may spend up to $90M/year, nearly their entire usage revenues, paying gas, trucks and employees to manually move bikes from full to empty stations (Fig. 1). Yet, many BSS suffer from low level of service. During survey administered by the French company TNS Sofres, 48% of users in Velib, Paris, were unsatisfied with bike availability and 58% complained of insufficient parking spaces at the stations [3]. Currently, in Barcelona, almost 50% of the stations are unavailable (empty or full) during 30% of the time [4]. In car sharing this can only be worse as employees must either tow or drive cars using other service vehicles to move between relocations. Despite those problems, BSS are rapidly expanding: as of today more than 200 new systems are planned across the world [5].

Figure 1. Truck carrying bicycles during redistribution, in Velib Paris

II. PRIOR WORK

BSS is a relatively new area in transportation literature. Some works focus on analyzing and predicting human mobility patterns from datasets [6]; other works focus on optimizing inventory rebalancing or modeling truck repositioning using stochastic [7] or deterministic [8] methods. Finding provably optimal repositioning solutions is practically intractable. To simplify the complexity of the problem, several works distinguish between stations that can meet their level of service requirements without inventory correction and stations that require inventory correction [9]. In reality, all stations end the day with some discrepancy that accumulates each new day, if not corrected. Other works distinguish between dynamic [during the day] versus static [during the night] repositioning [10]. In practice, most repositioning is done empirically using real time information from the stations, directions from a dispatcher, and the truck drivers' experience [11]. Even if routing methods were optimized the true cost of reposition would still remain prohibitively high. Considering the average lease of a sprinter truck, wage of a work shift, gas costs, and the maximum number of bikes that a truck with 25 slots repositions per day, a truck with 2 workers operating in 2 work shifts spends about $650 to reposition 140 bikes or $4.6 per bike repositioning, excluding IT costs. This cost ranges between $1.5-$15, depending on pricing, trip pattern characteristics, and routing efficiency.

Not surprisingly, many bike-sharing experts argue that the next generation of intelligent BSS will rely heavily on incentivizing user behavior to mitigate, or even eliminate, operation burden [1][2][12]. Several current approaches are worth mentioning: *Velib* in Paris, rewards 15 minutes of additional time to users who ride their bikes up to the elevated stations at hills of Paris, marked with the V+ sign [13]. *Capital Bikeshare* in Washington offers points to users that drive bikes from full to empty stations allowing them to earn prizes that extend their membership for free [14]. Similar approaches have been implemented in pilot car-sharing systems such as the *IntelliShare* in the University of California Riverside, that incentivized users to ride individually from full to empty stations while share rides from empty to full stations reducing towing trips of up to 43% [15]. One of the obscurities of such reward offerings is that there is no clear mechanism for evaluating them. How much should a reward be and where will the funds to pay it come from?

Markets are used for evaluating assets or services exchanged between suppliers and consumers such that in equilibrium each side is getting what is willing to give up for. Competitive markets have furthermore the property of maximizing the social welfare of their participants. Market mechanisms are often used in resolving allocation of scarce resources in networks with bandwidth and capacity constraints. Other benefits over traditional Operations Research methods include scalability, autonomy, and self-organization especially in larger scales. Applications include: file sharing systems [16]; train scheduling mechanisms [17]; computational grids and communication in sensor networks [18][19], *hidden markets* for P2P backup systems (systems where users participate without being aware of it) [20]; smart power grids and dynamic pricing in electricity grids; online display advertising; congestion pricing; electronic auctions; carbon trading programs; and water banking systems [21].

In transportation industry, market mechanisms have been used for preventing either bottlenecks at links or spillovers at nodes of networks. Interstate truck rentals use variable market-based pricing depending on the pick-up and drop-off locations. For example U-Haul, a truck rental company based in US, prices a trip from Detroit to Houston about 170% higher than the other way around [22]. Smart parking systems use dynamic pricing to regulate demand and supply. For example, San Francisco launched in 2011 the first dynamically priced public parking system in the US [23]. Nevertheless the objective of an operator in BSS is not to avoid bottlenecks or spillovers but rather to maximize circulation of vehicles given capacity constraints. Since a trip requires both a vehicle at an origin and a parking space at a destination, a pricing model should take into account availability of both.

We believe that the design of a pricing model for BSS should address three key issues. First, trip prices should reflect the demand and supply in both origin and destination stations. Second, the pricing model should channel revenues from penalties to finance rewards reflecting that high-payers pay in fact low-payers to reposition vehicles. Third, it should ensure that prices change slower than inventories to ensure system stability. The latency of the system depends on the average trip time and the system's size: for example, in small deployments such as a university campus, changes in price would instantly reflect on inventory levels. In contrast, in large deployments such as a metropolitan district, price changes would take hours to reflect on inventory levels.

The pricing model we describe in the next section is inspired by a rather unusual paradigm. Two-sided markets with intermediaries are institutional mechanisms consisting of buyers, sellers, and intermediary traders. Traders buy goods from competing sellers and resell them to buyers. Mutual peer competition, cost avoidance, and profit maximization, force prices to maximize the flow of goods from sellers to buyers, channeling it through the traders [25].

III. THE MARKET ECONOMY OF TRIPS

A. *Motivation*

The Market Economy of Trips (MET) is a self-balancing operation model for MoD systems that motivates users to rebalance the fleet using price incentives. Like a stock market, trip prices change dynamically based on inventory needs of origin and destination stations causing some trips to be more expensive while other trips to pay back money [24]. Stations 'bid' and 'ask' prices based on inventory needs and the competition with neighbor stations. Trip values are determined by the transactional difference between "buying" a vehicle from an origin and "reselling" it to a destination (Fig. 2). By redirecting funds from overpaying to underpaying users the system converges to a self-sustaining equilibrium.

If the pick-up price that an origin 'asks' is lower than the drop-off price that a destination 'bids' then the user wins the transactional difference from the system as a reward; in the opposite case the user pays the system the difference. Finally, if the pick-up price that the origin 'asks' is the same as the drop-off price that the destination 'bids' then the ride is free for the user. Thus the same drop-off location may be cheap for someone and expensive for someone else depending on where they come from. During pick-up, users lock prices for certain time period (e.g. 30min) and they pay during drop-off. The MET is a self-organizing system operated by and for its users.

We propose a graphic user interface (GUI) that uses a heat/contour map display to relate prices to slope gradients to communicate price information to the users (Fig. 4): isometric price curves describe areas with same price indexes. Like navigating through a price landscape, climbing from valleys up to hills is expensive, descending from hills down to valleys is rewarding, while traveling through flat areas is neutral.

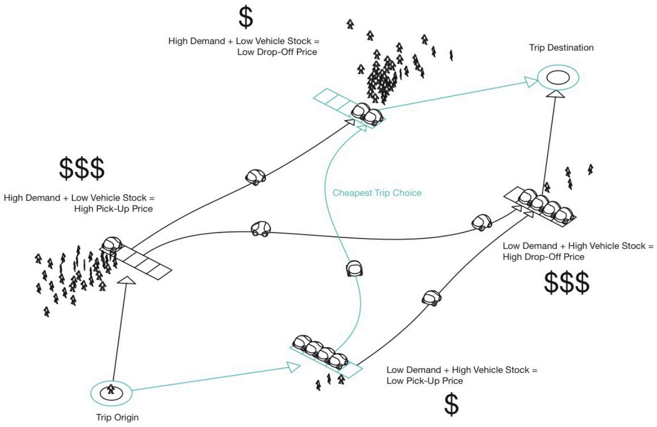

Figure 2. The origin-destination path (in blue) with highest payoffs in MET

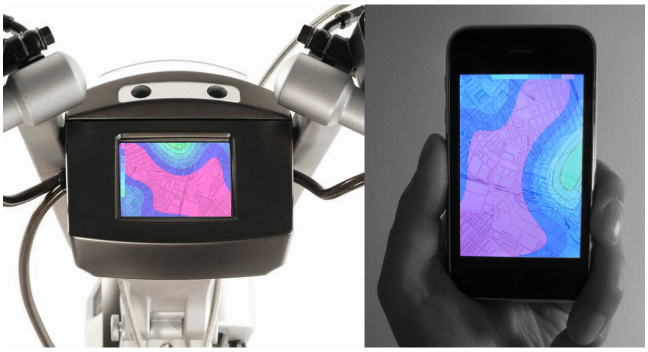

Figure 3. Contour-map GUI: a possible interface for communicating payoffs using isometric curves of price indexes

B. *Analysis of the MET as an efficient marketplace*

We define the utility of a trip as the total cost spent on fare and fuel minus the total cost of time spent on travelling based on the individual user's evaluation of time. On average, prices of urban mobility modes reflect the commuting times they take in relation to how their users evaluate their time. Commuters whose cost of time is high are willing to pay higher for faster options than commuters whose cost of time is low. Urban trips are combinations of multiple mobility modes: walk from house to a nearby bus station; take the bus to the downtown center; ride a bike inside the center; finally, walk again to the final destination. The commuting cost of each compound trip is the sum of the prices that have been paid for each option plus the total cost of time that was spent while traveling. People select those bundles of options that maximize their utility.

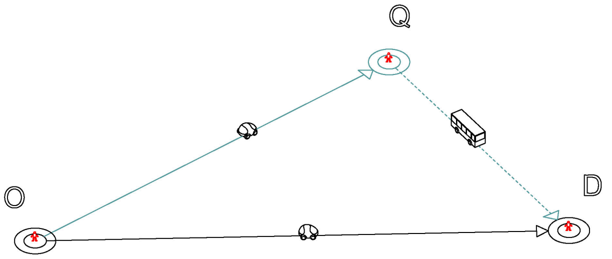

Figure 4. User decision making process. In equilibrium payoffs from OD should equal payoffs from OQ plus QD

The following analysis assumes that people think rationally and can efficiently perceive their payoffs. For the users the net result of a transaction should yield less commuting costs than using their best alternative option. For example, a user traveling from an origin O, and ends to a final destination D, will select any drop-off station Q if the price from O to Q with VSS plus the price from Q to D with the substitute (e.g. bus, taxi, walking, etc.), plus the total time costs (from O to Q with MOD, and from Q to D with substitute) are in sum less or equal than the original price from O to D with the substitute plus time cost with substitute (Fig. 4, Fig. 5). Users whose time cost is high will therefore be willing to pay higher prices in MET to minimize their total commuting time, while users whose time cost is low would be willing to spend more time in commuting for a better price in the MET. For the stations, the transactions should balance revenues from pick-ups with costs from drop-offs and capital expenses such as vehicles and urban land occupation, and yield a marginal profit.

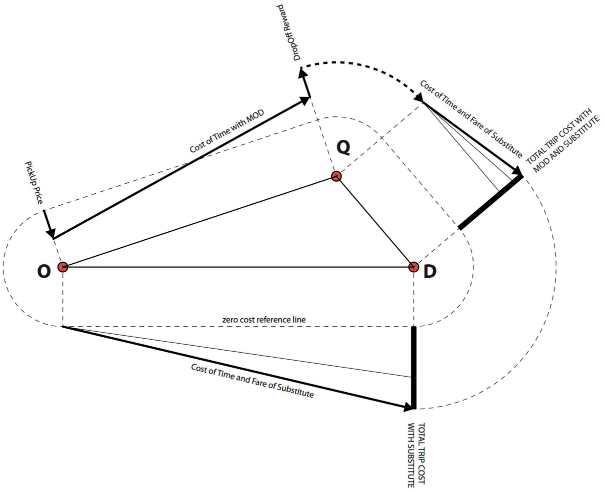

Figure 5. Price equilibrium graphic analysis of figure 5: The total trip cost with VSS from O to Q and with substitute from Q to D consist of: the downward pointing pick-up price vector at O; the up-warding inclined price vector from O to Q that depends on user's cost of time and VSS's traveling speed; the up-warding drop-off reward vector at Q; and the up-warding inclined vector from Q to D that depends on user's cost of time, substitute's traveling speed, and substitute's fare per distance traveled. The total cost is indicated by the thick line at D. Similarly, the total trip cost from O to D with the substitute consist of the up-warding inclined price vector from O to D that depends on user's cost of time, substitute's traveling speed, and substitute's fare per distance traveled.

The Market Economy of Trips is a form of a strategic game. Territorial decisions of users change the pricing of stations, which changes the payoff landscape affecting decision making of other users and vice versa. Urban Economic theory shows that users with sufficient information would make decisions that minimize their time-adjusted commuting costs bringing eventually the system into a competitive equilibrium where no further action can be taken to increase anyone's payoffs [26]. Such equilibrium would come when no high paying and no low paying users prefer any other resources at the prices, including not participating.

C. Convergence of the MET

To explain how the Market Economy of Trips converges from one equilibrium state to another, we illustrate a rather unusual case. Suppose the MET starts in maximal flow equilibrium. In this ideal state, average vehicle inflows and outflows are equal in each station and as a consequence all inventory levels remain unchanged. Furthermore since vehicle flows are maximal all inventory levels must be the same (maximal flow equilibrium). As trip pattern becomes imbalanced, stations with inventory shortages increase both 'ask' and 'bid' prices causing 'buying' a vehicle from there to be expensive, while 'selling' a vehicle there to be rewarding. Similarly, stations with inventory surpluses decrease both 'ask' and 'bid' prices causing 'buying' a vehicle from there to be cheap, while 'selling' a vehicle there to be non-worthy. As the gap between prices increases, moving from stations with shortages to stations with surpluses gets more expensive, while moving the other way around becomes profitable.

Commuters with low cost of time will be willing to pick-up from stations further from their true origins and drop-off to stations further from their true destinations as long as this is still cheaper for them than using the substitute mode. Similarly, commuters with high cost of time will be willing to pickup from expensive stations closer to their true origins and drop-off to stations closer to their true destinations as long as the net result is still cheaper for them than using the substitute. In the most imbalanced demand pattern, the cost that each user is paying is marginally the same as the cost he would be otherwise paying with the substitute mode. Any additional change from this point will cause some users to opt out. As some users respond to prices while others opt out, the trip pattern asymmetry decreases causing the gap between prices at each station to decrease. This in turn increases throughput performance bringing eventually the system into a new equilibrium state.

D. Monopoly versus competitive pricing

We observe two models for controlling pricing at the stations: monopoly and competitive pricing. In monopoly pricing, all stations represent one profit-maximizing firm and as a consequence they choose 'bid' and 'sell' prices that maximize revenues from departing trips and minimize costs for arriving trips; such setting however does not maximize vehicle flow in the system. In contrast, in competitive pricing all stations represent individual profit-maximizing traders that compete each other over prices: on one hand each station-trader tries to 'ask' a lower pick-up price than its nearest neighbor's 'ask price (but not lower than its own 'bid' price), and on the other hand it tries to 'bid' a higher drop-off price than its nearest neighbor's 'bid' price (but not higher than its own 'ask price). Through mutual competition with its closest neighbors, each station drives 'ask' prices down and 'bid' prices up (to attract more users) until they match, maximizing thus flow of vehicles.

E. The financial system of the MET

While the physical system is watertight (the vehicles can never escape the vehicle sharing system), the financial system is open (money flows in as revenues from users and flows out as costs for rewards). The financial system of MET can be studied in two levels. At the station level, the revenues are determined by the pick-up price multiplied by the departures rate, while its costs are determined by the drop-off reward multiplied by the arrivals rate, and the payments for capital expenditures. During convergence, stations are trying to balance revenues with costs by adjusting pick-up and drop-off prices to redirect revenues from departures to costs for rewarding arrivals. At the system level, the gross revenues come from the flow of high-paying trips multiplied by the difference between the pick-up price at net sources and drop-off reward at net sinks, while the gross costs go on financing the flow of redistributing sink-source trips plus capital expenditures. During convergence, the MET is trying to balance revenues with costs by adjusting trip prices to redirect revenues from expensive source-sink trips to costs for rewarding sink-source trips.

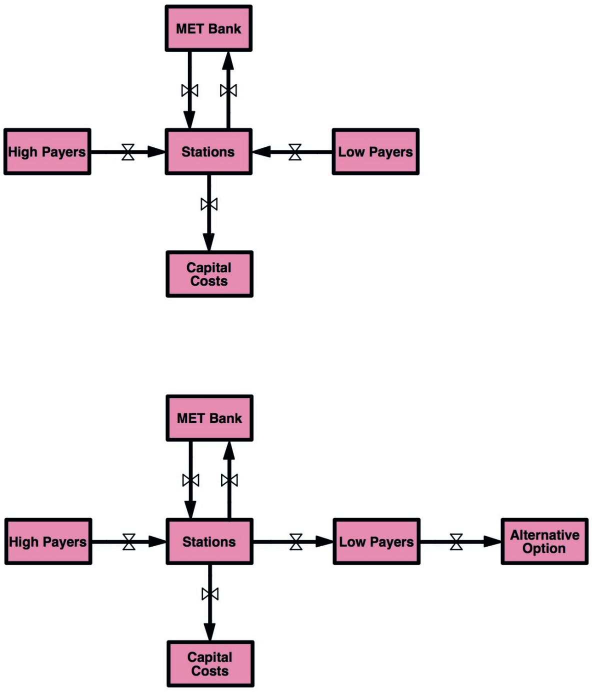

Figure 6. Cash flow system during financial disequilibrium and symmetric demand pattern (top), and asymmetric demand pattern (bottom)

While each station has a local balance account for revenues and costs, during operation it may enter in a state of either financial deficit or financial surplus. There are two main reasons for this. First, there is a delay between the impact of pricing on users and the effect of their actions at the stations, which mainly depends on the average travel time. Second, during pricing the local financial resources of the station may not be sufficient to provide the desired rewards to incentivize users which could potentially bring the performance of the station down to a vicious cycle: less available funds, less rewards offered to low-payers, less redistribution from low-payers, less demand from high-payers, less profits, and finally even less available financial resources. To avoid such circumstances, stations with financial deficit may need to borrow from stations with financial surplus. This fund reallocation must be carried out by a "central bank" and a borrowing/lending policy. Stations lend the bank during profit making borrowing from the bank during losses. The central bank thus provides a buffer that can sustain the system during demand fluctuations.

The above observations are illustrated in the two diagrams in figure 6. During symmetric trip patterns all pick-up and drop-off prices are equal and all users pay the same price (Fig. 6, top). During asymmetric trip patterns financial resources are redirected from high-paying users to low-paying users through the stations, which in turn are redirected from low-paying users to the substitute option (Fig. 6, bottom).

F. *Competitive equilibrium*

The following holds true during long-run equilibrium.

- The daily inbound and outbound volumes are equal (e.g. all stations end the day with the same inventories as they started the day).
- All users make a portion of their trip using a MoD vehicle and another portion using the substitute. The distance that the high-paying users travel with the substitute is on average less than the distance that the low-paying users travel with it.
- The total commuting costs that each of the two groups of payers pay consist of the price they pay for MoD as this is defined by the pick-up price and the drop-off reward, the price they pay for the substitute, the time cost they suffer by using a MoD vehicle and the time cost they suffer by using the substitute. Since the substitute is slower than the MoD vehicle its time costs are higher than those of the MoD vehicle.
- The cash flows that high-paying users pay to MET equal the cash flows that MET pays to the underpaying users plus the cash flows that MET pays for capital costs.
- The cash flows that high-paying users pay to MET equal the throughput rate times the difference between average pick-up prices at net source stations and average drop-off rewards at net sink stations.
- The cash flows that MET pays to the underpaying users equal the throughput rate times the difference between average pick-up prices at net sink stations and average drop-off rewards at net source stations.
- The cash flows that low-paying users pay to the substitute plus their total time costs (by both MoD and the substitute) minus the cash flows that they receive from MET are less or equal to the cash flows that they would otherwise have to pay to travel from sources to net sinks with the substitute plus their time costs with from the substitute
- Similarly, the cash flows that the high-paying users pay to MET plus the cash flows they pay to the substitute plus their total time costs (by both MoD and the substitute) are less or equal to the cash flows that they would otherwise have to pay to travel from net sources to net sinks with the substitute plus their time costs with from the substitute (which are much higher than those of the low-payers because high-payers have high time costs).

Therefore the distribution of the personal cost of time on the population of commuters determines the theoretical performance of MET as it indicates the likelihood that some commuters would be willing to buy the time of other commuters. In a population where everyone evaluates time in the same manner nobody would be willing to spend more to commute faster and consequently nobody could earn rewards to drive further.

IV. DISCUSSION

In this paper we presented the Market Economy of Trips, a new operation model for MoD systems (bikes, scooters, automobiles) based on a double auction market where cost-minimizing users are both buyers and sellers of trip rights while profit-maximizing stations are competing auctioneers that trade them. Trip rights are priced relatively to the inventory needs of origin and destination stations. This paper formulated MET as a game theoretic problem by describing the participants, their interests and their decision-making processes. The global goal of the system is to incentivize circulation throughput by inversely pricing both pick-up and drop off points. This can be done through the form of a two-sided market where self-interested intermediary stations have an incentive to increase throughput flow. The paper further presented a pricing model that reflects an important requirement for sustainability: the flow of the funds that the overpaying users pay the system should balance the flow of rewards that the system pays the underpaying users. The underpaying users in turn pay the alternative option as they redistribute the vehicles. This means that a dynamically priced MoD system must exist synergistically with existing public transit infrastructure.

V. FUTURE WORK

As part of the next steps of this work we want to assess the limits of efficiency of the MET comparatively to those of current truck repositioning methods, using bike sharing as a case study. The difficulty of this endeavor is two-fold: on one hand there are no reasons to believe that current truck repositioning is done in the most efficient way. On the other hand we are not aware of existing applications of market based pricing similar to the MET in vehicle sharing systems. Furthermore, the work we presented here is based on the efficient market hypothesis, e.g., that users are rational and that they have full access to information. In practice, neither of those two assumptions holds fully true. We are currently working on two directions. In the first direction we are building a computational simulation model using data from bike sharing systems, to analytically explore the limits of efficiency of truck repositioning under the constraint that ridership revenues cover repositioning costs. In the second direction, we are designing a game-based life experiment using a BSS as a living laboratory. Our future work will compare the empirical results of the experiment with the analytical results of the simulation model.

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